BORSA ITALIANA®

BItNOTES

N. 8 - MAY 2003

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Giovanni Petrella[a]

Abstract

This paper investigates whether euro area small capitalization stocks are an asset class, i.e., if investing in this category of stocks enhances portfolio's mean-variance characteristics. We perform regression-based tests for mean-variance spanning in order to detect the effect of euro area small caps on the minimum variance frontier, and apply different measures to assess the extent of diversification gains.

Empirical analysis shows that euro area small and mid cap stocks, as classified by size quartile and quintile rankings, arise as truly autonomous asset classes. This result is robust to different methodologies used to form size-based portfolios, and holds relative to both euro area large cap stocks and other international asset classes, US small capitalization stocks included.

We also investigate the source of the rejection of the spanning hypothesis. We find that both the tangency portfolio and the global minimum variance portfolio can be improved by the inclusion of euro area small and mid cap stocks. The evidence is stronger in favour of the enhancement of the tangency portfolio. This result is particularly important for investors with low risk aversion, as they would benefit more from the enlargement of the minimum variance frontier.

Keywords: asset allocation, portfolio diversification, small cap stocks
JEL Classification: G11, G15

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I am grateful to Alessandra Franzosi for insightful conversations and suggestions, Mario Anolli, Francesco Cesarini, and Luca Filippa for helpful comments. I thank Enrico Pellizzoni for providing me with data on companies listed on euro area exchanges.

Are euro area small caps an asset class? Evidence from mean-variance spanning tests

Giovanni Petrella[a]

Abstract

*Obiettivo del lavoro è verificare se i titoli a limitata capitalizzazione (*small cap*) dell'area euro rappresentano una classe di attività finanziarie (*asset class*). A tal fine si confrontano le proprietà media-varianza di portafogli costruiti sulla base di due insiemi di titoli: uno che non include i titoli* small cap *dell'area euro e un altro che li include. Le conseguenze sulla frontiera efficiente derivanti dall'introduzione in portafoglio di titoli* small cap *dell'area euro sono statisticamente verificate attraverso l'uso di test di* spanning.

I risultati dell'analisi empirica indicano che i titoli small *e* mid cap *dell'area euro, individuati attraverso la ripartizione del campione sia in quartili che in quintili di capitalizzazione, rappresentano* asset class *autonome. Questo risultato è robusto rispetto alle metodologie utilizzate per la costruzione dei portafogli ed è valido nei confronti sia dei titoli* large cap *dell'area euro che dei titoli* large *e* small cap *statunitensi. Ciò indica che i titoli* small cap *dell'area euro si comportano in maniera strutturalmente diversa anche rispetto a titoli di capitalizzazione omogenea negoziati sul mercato statunitense.*

Il lavoro analizza infine le cause dei risultati ottenuti, ricercando le fonti del rifiuto dell'ipotesi di spanning. *L'inclusione di titoli* small cap *migliora le proprietà media-varianza sia del portafoglio a varianza minima, sia - con maggiore significatività statistica - del portafoglio di tangenza. Quest'ultimo risultato è particolarmente rilevante per gli investitori con limitata avversione al rischio, in quanto essi potranno beneficiare maggiormente dall'ampliamento della frontiera efficiente.*

Keywords: asset allocation, portfolio diversification, small cap stocks
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Desidero ringraziare Alessandra Franzosi per le stimolanti conversazioni e i proficui suggerimenti, Mario Anolli, Francesco Cesarini e Luca Filippa per gli utili commenti a una versione precedente del lavoro. Ringrazio Enrico Pellizzoni per avermi fornito i dati sulle società quotate in alcuni mercati dell'area euro.

1. Introduction

Long term asset allocation is arguably the most important set of decisions that an investor can make.[1] The first step of the allocation process is the identification of the asset classes to be considered. Failing in the consideration of an asset class might lead to sub-optimal portfolio decisions, e.g. to construct portfolios unable to attain the maximum (minimum) return (risk) *per* unit of risk (return). Portfolio managers can achieve efficient diversification only by first identifying the full range of asset classes from which to choose.

Major asset classes, commonly used in the definition of portfolios strategic (i.e., long term) asset allocation, are domestic stocks, international stocks, government bonds, corporate bonds, and the risk-free asset. Small capitalization stocks are sometimes disregarded in the portfolio strategic allocation process. In this paper we focus on small capitalization stocks, and we look at the differential effect of investing in this category of asset. Thus, the basic question we address in this study is whether euro area small cap stocks are an asset class, i.e., if there is any benefit – from a portfolio efficiency perspective – in investing in euro area small caps.

An important issue for both individual and institutional investors concerns the existence and the magnitude of the benefits from diversifying over small capitalization stocks. Based on standard portfolio theory, if small cap returns do not perfectly correlate with large cap returns, investors gain from 'size diversification'.[2] A number of studies show that a difference in return behavior between small and large capitalization stocks exists (e.g., among others, Chan and Chen, 1991; Reinganum, 1992). This suggests that diversifying into small stocks might improve portfolio performance. However, the existence and the scale of size diversification benefits have been left to be properly investigated.

The issue of investing into small cap stocks is interesting for at least three reasons. First, as a group these stocks represent a non-negligible portion of the overall stock market. For the US market, considering a long term average, small capitalization stocks account for about 12 percent of the total market capitalization (Pradhuman, 2000). Second, if small cap stocks are an asset class they should be properly considered when defining portfolio strategic asset allocation to avoid sub-optimal decisions, and the definition of strategic asset allocation has important implications for the long-run investment return (see footnote 1). Third, little research has been done on the benefits of size diversification, especially for non-US markets. Here we focus on euro area stock markets.

The question we investigate in this paper is whether an investor can enhance the

(1) Ibbotson and Kaplan (2000) examine a sample of U.S. balanced mutual funds and pension funds, and find that about 90 percent of the variability in returns of a typical fund across time is explained by policy (i.e., long term asset allocation) decisions, about 40 percent of the variation of returns among funds is explained by policy, and on average about 100 percent of the actual return level is explained by policy return level.

(2) In this study 'size diversification' indicates an asset allocation strategy that invests in various market capitalization-ranked stock classes.

mean-variance characteristics of her portfolio by investing in euro area small cap stocks. Specifically, the objective of the study is to investigate statistical significance and magnitude of diversification benefits arising from investing in euro area small cap stocks. We apply a mean-variance spanning approach to test for the existence of such benefits, and estimate the dimension of diversification gains using several empirical methodologies.

Mean-variance spanning tests have been previously applied mainly to investigate the benefits from international diversification. Testing for spanning means to test whether adding a 'new' set of risky assets, i.e., small caps in our study, allows investors to improve the minimum variance frontier derived from a given set of risky assets. Alternatively stated, mean-variance spanning is a test of the hypothesis that investing in small caps enlarges the efficient frontier. The spanning hypothesis is rejected when the 'old' assets are not able to replicate the mean-variance characteristics of the new assets under investigation and thus the former can be considered as autonomous asset classes.

Results indicate that euro area small and mid cap stocks, as classified by size quartile and quintile rankings, arise as truly autonomous asset classes. This evidence is robust to different methodologies used to form size-based portfolios and different sets of benchmark assets. This result holds relative to both euro area large cap stocks and other international asset classes, US small capitalization stocks included. We also investigate the source of the rejection of the spanning hypothesis. We find that both the tangency portfolio and the global minimum variance portfolio can be improved by the inclusion of euro area small and mid cap stocks. The evidence is stronger in favor of the enhancement of the tangency portfolio. This result is particularly important for investors with low risk aversion, as they would benefit more from the enlargement of the minimum variance frontier.

The paper is organized as follows. Section 2 provides a brief summary of the main empirical evidence on small cap returns. Section 3 illustrates the methodology used in this paper to estimate existence and extent of diversification gains. Section 4 describes the data and Section 5 discusses empirical results. Conclusions and implications are presented in Section 6.

2. A review of the evidence on small cap returns

In this Section we provide a summary of the main empirical evidence on small cap returns.[3] First academic studies dealing with small caps date back to the eighties, when Banz (1981) reported that, over long investment horizons, small cap stocks had substantially higher returns compared to large cap stocks even after controlling for estimated betas, that is after adjusting returns for market risk. This paper triggered a number of subsequent studies to explain the reasons of what was called the 'size anomaly',[4] because of the fact that this evidence could not be reconciled at that time within the extant theoretical pricing models.[5]

Stoll and Whaley (1983) suggested that transaction costs could, at least partially, account for the abnormal risk-adjusted returns earned on small cap stocks. In fact, they found that, after adjusting for both trading costs and market risk, small stocks earn lower returns than large stocks if bought and held for two months or less. As the investment horizon increases, the after-trading costs abnormal return for small caps does become positive. Thus, when answering the question whether an investor can earn positive abnormal returns net of transaction costs by trading on the basis of stock market value, they state that "the answer to the question is contingent upon the length of the investment horizon." That is, investor holding-period plays a central role in determining the profitability of a portfolio strategy based on small cap stocks. And this has important practical implications for portfolio managers investing in small cap stocks.

More recent studies cast a shadow on the validity of the first results on size anomaly. Berk (1996) still finds that the returns of quintiles sorted on market value monotonically increase as the size of the quintile decreases, however he fails to find a significant relationship between average returns and four other, non-market based, measures of firm size (book value of assets; book value of property, plant, and equipment; total value of annual sales; total number of employees). Horowitz et al. (1996, 2000) investigate the relationship between stock returns and firm size for data from NYSE, Amex and NASDAQ in the period 1980-1996, and find no evidence of size premium.[6] There appears to be, on the sample period, a negative size premium; that is, large firms have slightly higher realized returns than small firms do. Dimson and Marsh (1999), using data on both US and UK markets, also suggest that the size premium has disappeared, and that perhaps it may have gone into reverse. Indeed, based on data up to 1997, they find that large cap stocks appear to have higher returns than small cap stocks do. Reilly and Wright (2002) update the analysis of small caps performance through the year 2000 and

(3) Levis (2002) provides a comprehensive review of the evidence on small caps with a special focus on the UK market.

(4) A set of papers that summarizes the original debate on size anomaly can be found in the Special Issue "Symposium on size and stock returns, and other empirical regularities" of the *Journal of Financial Economics* (1983, vol. 12, no. 1).

(5) Banz's evidence was inconsistent with the CAPM, and established that beta-based benchmarks could be beaten by adopting size-based strategies.

(6) Size premium (or small cap premium) is the difference in returns between a portfolio of small cap stocks and a portfolio of large cap stocks. To identify small and large stocks, academic researchers often refer to quintile or decile portfolios. That is, the universe of securities is broken into five (quintile) or ten (decile) portfolios, each with the same number of securities.

also find a reverse small cap effect, i.e. large cap stocks outperform small cap stocks over the period 1984-2000.

Such contrasting evidence can be interpreted on the basis of the hypothesis of cyclical behavior of the size premium. Brown et al. (1983) initially suggested the variability over time of the size premium. They investigated and rejected the hypothesis of stability of excess returns obtained by ranking stocks according to their market value; over some periods small caps outperformed large caps, while in other periods the effect was reversed. More recently, Reinganum (1992) reports that the variability of the size premium is not entirely random; rather, the size premium reverses over long time horizons, and exhibits almost predictable patterns. Specifically, Reinganum collects data on US markets for a long time span (1926-1989) and computes rolling (partially overlapping) portfolio returns compounded over different investment horizons. Next, he computes the size premium as the difference between a small cap portfolio return and a large cap portfolio return, adopting several definitions of small and large caps. Lastly, he estimates portfolio returns autocorrelation, i.e. the correlation between current returns and past returns, for investment horizons ranging from one through seven years.

Reinganum's empirical results show that return autocorrelation is positive over one-year and two-year investment horizons, and becomes negative for investment horizons lasting longer than two years. In particular, return autocorrelation is negative and statistically significant at 5% level for five-, six-, and seven-year horizons; the six-year horizon exhibits the lowest autocorrelation and the highest statistical significance. As a practical implication, this negative autocorrelation means that the size premium tends to reverse over longer investment horizons: periods in which the size premium is positive tend to be followed by periods in which the size premium is negative.[7]

As Levis (2002) points out, this cyclical pattern raises the question of whether the size premium may be driven by economic fundamentals. Specifically, the behavior of different size stocks could be related to different reactions of the cash flows and discount rates of such firms to changes in economic conditions. Chan et al. (1985), in an attempt to explain the size premium in a multi-factor pricing model framework, study the sensitivity of stock returns to different risk factors. They find that a large portion of the size premium is explained by changes in the risk premium, and this factor is in turn related with changing business conditions. Thus, Chan et al.'s evidence suggests that small firms fluctuate more with economic expansions and contractions. Along the same line of research, Jensen et al. (1997, 1998) specifically relate the size premium to the monetary policy conditions in the US. They also observe, as noted earlier, that the small cap premium varies over time and, after controlling for the changes in monetary conditions, find that the size premium is positive and significant in periods of expansive monetary policy, while insignificant or negative when monetary policy is in a restrictive stance.

(7) Grieb and Reyes (2002), based on UK data, analyze the temporal relationship between large and small cap returns. They find that shocks to the large cap index positively affect its next period correlation with the small cap index, whereas shocks to the small cap index negatively affect its next period correlation with the large cap index. This evidence is also consistent with the cyclical behavior of the size effect.

Levis and Steliaros (1999) provide a detailed investigation of the performance and key characteristics of small cap stocks across seven European countries. Consistently with previous studies, they also find, *inter alia,* that small caps appear to be the first affected by turns of the business cycle.

The results of the previous studies call for a structural relationship between small caps performance and macroeconomic conditions. Indeed, these papers show that small and large firms react differently to economic swings, but they do not suggest why. Chan and Chen (1991) examine differences in structural characteristics that may lead firms of various sizes to react differently to the same economic news. They argue that a small cap portfolio contains a large proportion of what they call 'marginal firms', that is firms that possess one or more of the following characteristics: high financial leverage, cash flow problems, or low production efficiency. Such firms may suffer from limited access to external financing, especially during tight credit periods. Since a small cap portfolio tends to contain a higher number of 'marginal firms' – in Chan and Chen's meaning – than a large cap portfolio does, small cap returns react differently from large cap returns to the same piece of macroeconomic news.

Another line of explanations relates the differences in return of small and large companies, that is the 'size effect',[8] to the underlying fundamentals and market characteristics of small firms. Dimson and Marsh (1999) and Levis (2002) present several empirical findings that highlight the main differential characteristics of small caps and validate the hypothesis that the size factor in returns is related to the existence of a size factor in fundamentals. We briefly summarize three of those findings in what follows. First, Levis points out that smaller firms rely more on short term financing: the average ratio of short term loans to assets decreases monotonically with firm size. This fact, which is also related to monetary policy conditions, implies that small caps suffer more when credit market conditions worsen.

The second finding deals with the relationship between small cap returns and industry performance. The uneven industrial distribution of small and large companies could in principle result in sector-related performance differences. Stated differently, given that smaller companies are concentrated in specific industrial sectors, small cap returns might depend on relative sector performance. Dimson and Marsh (1999) re-compute the size premium by applying the sector weighting scheme of a small cap index to the calculation of a large cap index and find that a sizable portion of the size premium is explained by differences in sector weightings. However, the weights used to compute large cap returns are only theoretical. Thus, Levis (2002) compares portfolios of small and large cap stocks, actually listed, belonging to the same sector. He concludes that the industrial performance contributes to the size premium, but it does not determine it.

The third explanation of the difference between small cap and large cap returns looks at the dividend performance of small caps. Dimson and Marsh (1999) find that large

(8) The term 'size effect' is related, albeit not equivalent to, the term 'size premium'. Following Dimson and Marsh (1999), size premium indicates small caps outperformance relative to large caps, while size effect only refers to "the tendency for small cap stocks to perform *differently* from large cap stocks" (p. 65, emphasis in the original).

part of the performance differential between small and large cap can be explained by differences in dividend growth. On this point, Levis (2002) also argues that the superior earnings growth by small firms is predominantly due to the extraordinary growth of some companies in this group, rather than to the universal faster growth record of such firms. He also produces evidence that the standard deviation of earnings growth for small cap stocks is almost twice as large as that computed for large cap stocks. This implies that it is much more difficult to predict performance of small caps than performance of large caps.

The evidence summarized in this Section indicates that small cap returns tend to behave differently from large cap returns, i.e., a size effect exists, and that small caps distinguish themselves in force of specific both economic- and market-related characteristics. More generally, this evidence suggests that size-based portfolios could, at least in principle, perform according to structurally different pattern and, thus, form the basis for portfolio allocation. In the rest of the paper we investigate the hypothesis of size-sorted portfolios as autonomous asset classes and baseline inputs for strategic asset allocation.[9]

(9) Previous studies that highlight the importance of market capitalization in portfolio management are Reinganum (1983, 1999), Copeland and Copeland (1999).

3. Methodology: testing for and measuring diversification gains

There are two questions that, although strictly related, need to be separated in order to properly address the issue under investigation. First, we need to know if size diversification benefits exist (sub-Section 3.1) and, secondly, if existing, we need to assess the magnitude of such benefits (sub-Section 3.2).

3.1 Testing for the existence of diversification benefits

The addition of new assets to a portfolio made of a finite number of securities will in general cause a shift in the estimated mean-variance frontier and the estimated volatility bound. However, this shift may very well be the result of estimation error. The first question to address thus becomes whether the observed shift has to be attributed to chance or is a permanent modification. To answer the question whether or not the observed shift in the mean-variance frontier is significant in statistical terms, in this sub-Section we will show how regression analysis can be used to test for spanning and intersection.

The methodology we apply in this study was first developed by Huberman and Kandel (1987) to statistically test whether adding a new set of risky assets allows investors to improve the minimum variance frontier derived from a given set of risky assets. This test is usually referred to as 'spanning test', the additional assets as 'test assets' or 'new assets', and the assets belonging to the initial set as 'benchmark assets'. Given this terminology, the concept of spanning can be expressed as follows: we say a set of K risky assets (benchmark assets) spans a larger set of K + N risky assets (benchmark and test assets) if the minimum variance frontier generated by the K assets is identical to the minimum variance frontier generated by the K assets plus N additional assets.

Huberman and Kandel (1987) propose a mean-variance spanning test developed in a regression-based framework. Assume the following linear model:

$$\underline{r} = \underline{a} + \mathrm{B}\underline{R} + \underline{e} \qquad [1]$$

where $\underline{r}$, $\underline{a}$, and $\underline{e}$ are $\mathrm{N} \times 1$ vectors, $\underline{R}$ is a $\mathrm{K} \times 1$ vector, and B is an $\mathrm{N} \times \mathrm{K}$ matrix. The vectors $\underline{r}$, , $\underline{R}$ and $\underline{e}$ are random. The random vector $\underline{e}$ is uncorrelated with the random vector $\underline{R}$, and the expected value of each element of $\underline{e}$ is 0.

To assess whether the minimum variance frontier derived by $\underline{R}$ is the same as the minimum variance frontier generated by $\underline{R}$ and $\underline{r}$, we need to test the following two relations:

$$\underline{a} = \underline{0}_N \qquad [2a]$$

and

$$\mathrm{B}\underline{\iota_K} = \underline{\iota_N} \qquad [2b]$$

where $\underline{0}$ ($\underline{i}$) is a vector with elements all equal to 0 (1). The subscript indicates the dimension of the vector.

Restrictions [*2a*] and [*2b*] have an intuitive meaning: up to a zero mean, orthogonal factor, the returns on the N (new) test assets can be perfectly mimicked ('spanned') by a portfolio of the benchmark K assets. That is, each return of the additional assets can be written as a linear combination of the benchmark assets plus a zero mean error term.

Consider, as an example, an initial investment universe with three assets ($\mathrm{K} = 3$), and two additional assets ($N=2$) to be added to the original investment universe. Equation [1] would assume the following representation:

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

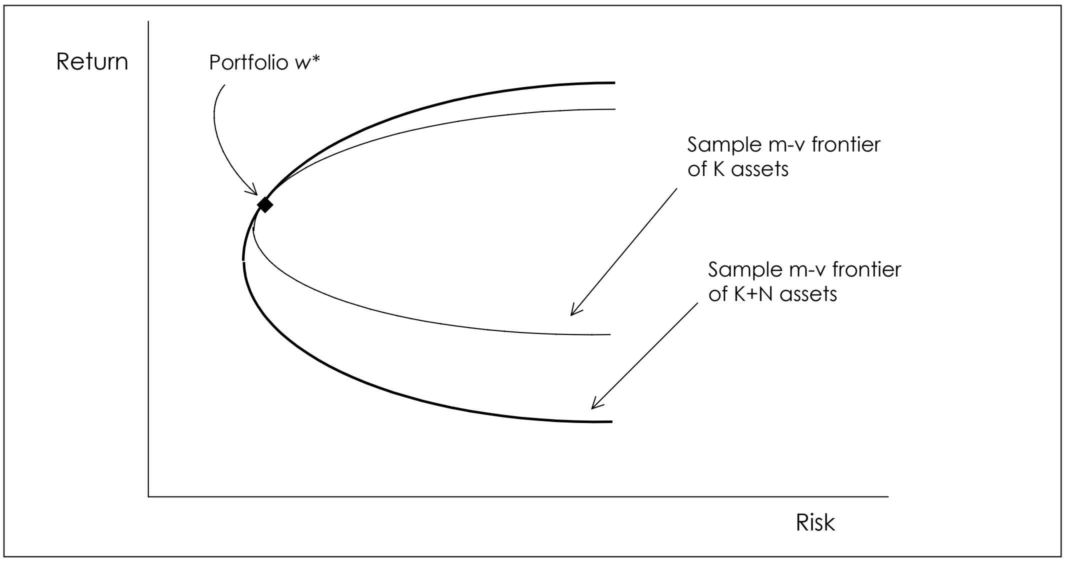
In this representation spanning implies that the returns on $N=2$ additional assets can be artificially reproduced by using the $K=3$ original assets.

In general, spanning refers to the case where the mean-variance frontier generated by an expanded investment opportunity set of benchmark assets and additional assets coincides with the frontier generated by the subset of benchmark assets only. The spanning test, therefore, answers the question whether the frontier shifts at any point when we add new assets to the investment universe. If there is spanning, no investor can improve her mean-variance efficient portfolio by including the new assets in her investment universe.

Related to spanning is intersection. Intersection refers to the case where the two frontiers intersect at a single point only (see Figure 1) or, alternatively stated, that there is a portfolio w^* which is mean-variance efficient for the original investment universe (K assets) and which is also mean-variance efficient for the larger investment universe ($K+N$).[10] This means that there is one mean-variance utility function (or risk-aversion parameter) for which there is no benefit from adding new assets. However, utility functional forms are usually unknown. Thus, in this paper we test for spanning that, by definition, is the situation when intersection holds for all risk-aversion parameters.

Figure 1 - Intersection

This graph plots sample minimum variance frontiers of K benchmark assets and K+N benchmark and test assets. The marker shows the intersection point.



(10) Spanning and intersection also have a dual interpretation in terms of volatility bound. Volatility bound is the lower limit of the variance of the stochastic discount factor derived in an intertemporal consumption and portfolio optimization problem. See, e.g., de Roon and Nijman (2001).

3.2 Measurement of diversification benefits

Several methodologies can be used to assess the extent of diversification gains. In this paper we apply three measures of portfolio efficiency as proxies for diversification gains.[11]

A first intuitive measure of diversification benefit is the reduction in portfolio risk achieved through the investment in the new asset class(es). Specifically, we calculate the difference between the global minimum variance (GMV) portfolio computed for the benchmark assets and the GMV portfolio computed for the benchmark and the additional asset(s). Thus, this measure accounts for the reduction, if existing, in portfolio risk that can be ascribed to the asset(s) under investigation.

To formally derive the first and the other two measures of diversification benefits used in this study we need some additional definitions and notations. We assume that the vector of the $K+N$ returns previously defined ($\underline{R}$ and $\underline{r}$) has a multivariate normal distribution, with mean $\underline{\mu}_{K+N} = [\mu_1,..,\mu_K,..,\mu_{K+N}]'$ and covariance matrix Ω_{K+N}. Let S be the set of all the real vectors $\underline{w}_{K+N} = [w_1,..,w_K,..,w_{K+N}]'$ such that $\sum_{i=1}^{K+N} w_i = 1$. The vector of weights of a portfolio can be thought of as a point in S. A set of constraints on portfolio weights is represented by a closed convex subset C of S. Thus, in our framework, C indicates the subset made only of K benchmark assets, i.e., $C = \{w_C \in S : w_i = 0, K < i \leq K+N\}$.

Given the above, a measure of diversification benefits that considers the effect on the GMV portfolio is the following:

$$\psi \equiv \min_w \left\{ \sqrt{\underline{w}_C{}'\Omega \underline{w}_C} \,\middle|\, w_C \in C \right\} - \min_w \left\{ \sqrt{\underline{w}_S{}'\Omega \underline{w}_S} \,\middle|\, w_S \in S \right\} \quad [3]$$

Related to this metric, two remarks are in order. First, note that ψ is positive when the new asset induces a reduction in GMV portfolio riskiness. Second, with this measure we implicitly assume that investors are interested solely in minimizing risk and do not care about returns. Albeit this assumption is clearly strong, the interest in such a measure arises from the fact that it is independent from expected return estimation and it is well known that expected returns are more difficult to estimate than variance (Elton and Gruber, 1995).

A second measure of diversification benefit can be computed by using the Sharpe ratio. Modern portfolio theory states that, where there exists a risk-free asset and unlimited lending and borrowing at the risk-free rate is allowed, then investors who care only about the mean and variance of their portfolios will only be interested in the tangency portfolio of the risky asset (i.e., the one that maximizes the Sharpe ratio). In that case, investors are only concerned with whether the tangency portfolio from using K benchmark assets is the same as the one from using all K+N risky assets. Therefore, in order to assess the magnitude of diversification benefit, we compute the change in the Sharpe ratio of the tangency portfolio when adding the test assets to the set of benchmark assets (Bekaert and Urias, 1996).[12]

(11) Li et al. (2003) provide a detailed exposition of diversification benefits measures.

(12) Instead of using the change in the maximum Sharpe ratio one could alternatively use the Jensen's alpha as a measure of portfolio efficiency (de Roon and Nijman, 2001; Gerard, Hillion and de Roon, 2002).

The Sharpe ratio of the tangency portfolio gives the *largest* mean return per unit of standard-deviation risk attainable for the assets under investigation:

$$\lambda \equiv \max_{w}\left\{\frac{\underline{w}_S'\underline{\mu}}{\sqrt{\underline{w}_S'\Omega\underline{w}_S}}\middle| w_S \in S\right\} - \max_{w}\left\{\frac{\underline{w}_C'\underline{\mu}}{\sqrt{\underline{w}_C'\Omega\underline{w}_C}}\middle| w_C \in C\right\} \qquad [4]$$

The change in the Sharpe ratio measures the economic importance of the shift in the efficient frontier. A difference between the Sharpe ratios computed for the K benchmark assets and the K+N assets indicates that investors can enhance their returns *per* unit of risk by investing in the additional N assets.

The third way to measure the benefit arising from diversification is to compute the gain in expected returns obtained by moving from an efficient portfolio made of only K benchmark assets to an efficient portfolio derived from K+N assets, with both efficient portfolios having the same variance. The difference in expected returns on the two portfolios measures the magnitude of diversification benefits. In our analysis, this means that the benefits of investing in small caps can be expressed in terms of return increments over portfolios with the same level of risk that only include large cap stocks.

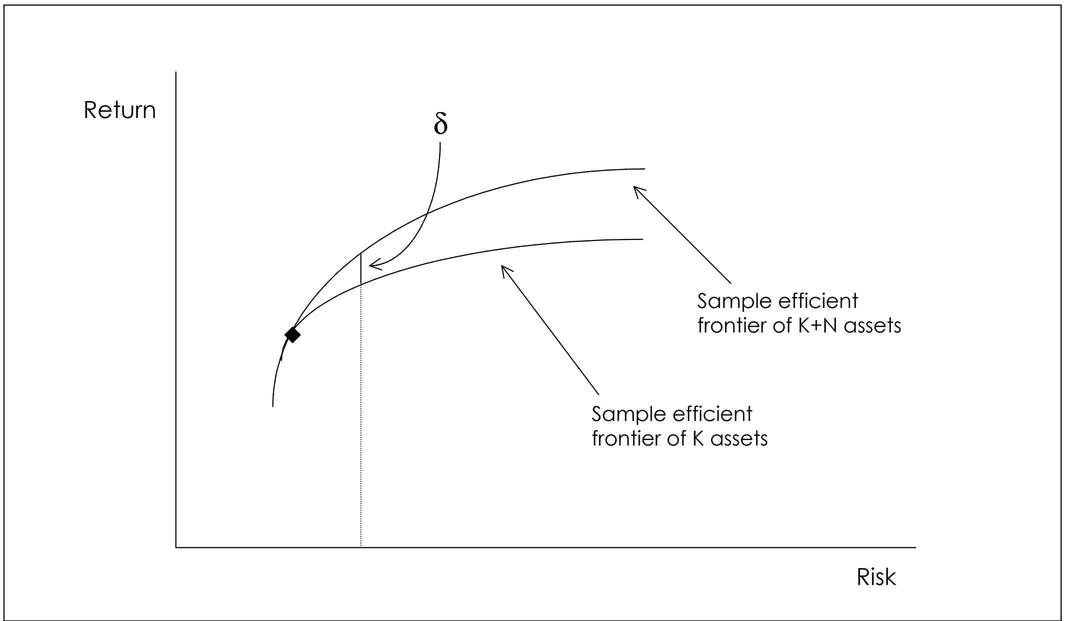
Given the framework defined above, an intuitive measure of diversification benefits that accounts for the return differential achievable holding risk constant is the following:

$$\delta \equiv \left\{\max_{w} \underline{w}_S'\underline{\mu} - \max_{w} \underline{w}_C'\underline{\mu} \middle| w_S \in S, w_C \in C, \underline{w}_S'\Omega\underline{w}_S \leq \underline{w}_C'\Omega\underline{w}_C\right\} \qquad [5]$$

A graphical representation of equation [5] is provided in Figure 2. This measure is similar in the spirit to that proposed by Wang (1998). However, differently from Wang, we consider several benchmarks to evaluate portfolio efficiency. In short, this is a generalization of Wang (1998) measure.

Figure 2 - Measuring diversification benefits

This figure provides graphical interpretation of equation [5].



In the same spirit, diversification benefits can be alternatively measured by the reduction in portfolio variance when investing in the new assets. In this case we need to estimate how much adding an additional asset can reduce the variance of a portfolio without changing its expected return. The extent to which the variance can be reduced depends on the variance of the new asset returns, and on its correlation with other asset classes.

A measure of diversification that considers the distance in terms of standard deviation between the two efficient frontiers is the following:

$$\phi \equiv \left\{ \min_{w} \sqrt{\underline{w}_S{}'\Omega\underline{w}_S} - \min_{w} \sqrt{\underline{w}_C{}'\Omega\underline{w}_C} \middle| w_S \in S, w_C \in C, \underline{w}_S{}'\underline{\mu} \geq \underline{w}_C{}'\underline{\mu} \right\} \quad [6]$$

Clearly, there is a straightforward relation between the two previous measures: ϕ is zero if and only if δ is zero. In this meaning, they are equivalent measures. Therefore, in the empirical analysis we only report results for δ.

4. Data description

In this study we investigate the effect on portfolio efficiency of the introduction of euro area small caps as an additional asset class. To test for the existence and to assess the significance of size diversification benefits, first and foremost we need time-series of returns for euro area large and small cap stocks that satisfy the following conditions: both large cap and small cap indices should be assembled using the same methodology, with no (or limited) overlapping between constituents, and available in total return index form.

We looked for return series satisfying the previous requirements within publicly available indices. However, we were unable to find a family of indices that appropriately fits into our analysis and thus we ended up in constructing our own euro area small, mid and large cap indices.[13,14] We considered all stocks listed on the four largest euro area exchanges in terms of market capitalization (France, Germany, Italy, and Spain)[15] and collected monthly stock prices from December 31, 1998 to December 31, 2002. We used monthly prices because using a shorter sampling frequency (e.g., daily prices) to compute returns introduces noise. This noise could make individual stocks (and portfolios) to look more independent than they truly are. Monthly returns are in fact less sensitive than daily returns to the biases from bid-ask bounce and thin trading effects (Ferson et al., 1993).

The sample period includes both a bull and a bear stock market cycle. The starting point coincides with the introduction of the euro currency (i.e., the fixing of parity), and was selected to avoid structural breaks in time-series data and to prevent currency volatility from influencing the optimization output. Exposure to currency risk is in fact a major determinant of international equity returns (Dumas and Solnik, 1995; De Santis and Gerard, 1998). Thus, we use post euro introduction returns in order to avoid currency risk effects. The sample does not suffer from survivorship bias as delisted stocks have also been considered in the analysis. Specifically, firms that are delisted either for good (e.g., takeovers) or bad (e.g., bankruptcies) reasons are included in the results until their particular delisting month.

We compute the monthly percentage return[16] for individual stock i in month t ($R_{i,t}$) as:

$$R_{i,t} = \frac{P_{i,t} + D_{i,t}}{P_{i,t-1}} \quad [7]$$

(13) We first referred to MSCI indices. However, when looking at indices constituents, we found significant overlapping between MSCI EMU Index and MSCI EMU Small Cap Index due to the fact that the former includes large, mid-size and small companies. Moreover, they are available as return indices only since January 2001 and this would clearly (strongly) restrict our sample period. Thus, we turned to Dow Jones STOXX indices, that are available as size indices for euro area (DJ Euro STOXX TMI Large, Mid, Small), but we were only able to find them in price index form. Salomon Smith Barney World Equity Indices are available as return indices, save for euro area stocks. We found the HSBC Smaller European Companies index available for euro area stocks and in total return form. However, HSBC does not provide a large cap index build upon the same methodology as the small cap version.

(14) Small, mid, and large cap return series used in this study are freely available from the author upon request.

(15) Based on FESE (Federation of European Stock Exchanges) statistics, those markets represent, in terms of market capitalization as of October 2002, about 74 percent of the aggregate euro-12 markets.

(16) We do not use continuously compounded returns (or log returns) for individual stocks since portfolio continuously compounded returns are not equal to the weighted average of the portfolio constituents log returns (Campbell et al., 1997, p. 11).

where $P_{i,t}$ ($P_{i,t-1}$) is the closing price for stock i at the end of month t ($t-1$) and $D_{i,t}$ is the dividend paid by stock i in month t.[17]

We construct size portfolios based on three methodologies: threshold approach, size quartiles, and size quintiles. While quartile- and quintile-based rankings are the norm for academic studies, the threshold approach has been selected as market professionals usually define small caps by using some threshold values of market capitalization, e.g. companies with a market capitalization under \$1 billion (Graja and Ungar, 1999, p. 20) or under \$1.5 billion (Pradhuman, 2000, p. 3). We use the lower value, as increasing the upper limit would enlarge the number of securities included in the portfolio and introduce noise in risk estimation, as will be shown later. Thus, in the threshold approach, stocks with market capitalization lower than or equal to euro 1,000 million are defined as 'small cap', stocks with market capitalization greater than euro 1,000 million and lower than or equal to euro 3,500 million are defined as 'mid cap', and stocks with market capitalization greater than euro 3,500 million are defined as 'large cap'.

Quartile- and quintile-based portfolios are formed as follows. First, all the stocks in the sample are sorted in order of ascending market capitalization at the end of each calendar year (December 31 of year t). Then, each stock is assigned to the proper percentile. Lastly, we form quartile and quintile portfolios in effect for the subsequent calendar year ($t+1$). Portfolios are rebalanced on a yearly basis.

Table 1 shows summary statistics for all the portfolios used in the empirical analysis. Data refer to December 31, 2001, i.e., this is the breakdown used to form 2002's portfolios. It is important to consider that the figures shown in Table 1 reflect the two-year bear market in effect since 2000, and this significantly affects the size of the stocks included in the portfolios. When we re-compute the same statistics as of December 31, 1998, the relative size of the stocks included in the sample increases: the largest stock in the third quartile has a market capitalization of euro 407.2 million, while it is euro 286.9 million in Table 1, and the largest stock in the third quintile has a market capitalization of euro 148.0 million, relative to 101.5 million in Table 1. This evidence is only the result of adopting an ordinally-based method to classify stocks according to their size.

(17) Closing prices are adjusted for corporate actions (e.g., stock splits, right issues).

Table 1 - Summary statistics for size-based portfolios

This table reports descriptive statistics for size-based portfolios. Portfolios are formed using three methodologies: threshold approach (Panel A), size quartiles (Panel B), and size quintiles (Panel C). In the threshold approach, stocks with market capitalization lower than or equal to euro 1,000 million are defined as 'small cap', stocks with market capitalization greater than euro 1,000 million and lower than or equal to euro 3,500 million are defined as 'mid cap', and stocks with market capitalization greater than euro 3,500 million are defined as 'large cap'. Size quartiles and quintiles are determined by sorting all stocks in order of ascending market capitalization at the end of December of each year. Monetary figures shown in the table are in millions of euros and refers to values as of December 31, 2001.

	N. of stocks	Market capitalization (millions of euro)				
		Average size	Standard deviation	Smallest stock	Median stock	Largest stock
Panel A: Threshold approach						
Small cap	2 232	124.8	194.5	0.1	40.0	999.6
Mid cap	189	1 845.3	643.6	1 004.3	1 688.8	3 416.8
Large cap	159	17 593.8	19 349.4	3 520.8	9 146.4	113 097.6
Panel B: Size quartiles						
1 (Smallest)	645	6.2	3.9	0.1	6.1	13.7
2	645	31.3	18.1	13.7	28.6	56.7
3	645	135.2	62.5	57.1	121.7	286.9
4 (Largest)	645	5 079.0	11 947.3	287.7	1 144.0	113 097.6
Panel C: Size quintiles						
1 (Smallest)	516	4.8	3.0	0.1	4.9	10.1
2	516	20.2	16.8	10.1	18.6	32.0
3	516	60.2	19.9	32.3	56.6	101.5
4	516	221.3	96.2	101.7	193.9	453.4
5 (Largest)	516	6 276.9	13 112.7	453.5	1 618.9	113 097.6

5. Results

Table 2 reports monthly return characteristics for portfolios categorized by size on the basis of the three approaches previously defined. Size-based portfolio returns display an interesting pattern through our sample period. The rates of return of the portfolios differ in a systematic way depending upon their market capitalization: the smaller the capitalization, the larger the return, no matter the approach used to form portfolios.[18]

However, time-series arithmetic averages of returns should not be interpreted as returns actually achievable over an investment horizon equal to the period used to compute such averages. That means, in our case, that these figures do not represent portfolio returns over a 4-year horizon. Arithmetic averages computed from monthly returns only represent returns for a 1-month holding period. To obtain estimates of returns for a 4-year horizon, geometric averages need to be computed (see, e.g., Elton et al., 1987). In Table 2 we also present geometric averages of time-series returns. We obtain similar results. Again, as for the arithmetic averages, we find that returns increase with decreasing market capitalization.

Risk, as proxied by return's standard deviation, is the highest for the smallest size portfolio, and tends to decrease as size increases. This pattern is only reversed for the threshold approach due to a computational bias that implies underestimation of risk for the smallest firms' portfolio. In fact, while quartile- and quintile-based portfolios include the same number of securities and inter-quartile or inter-quintile comparisons are meaningful, portfolios constructed according to the threshold approach do not contain the same number of constituents. Given that the dispersion of portfolio returns is inversely related to the number of stocks in a portfolio, it is meaningless to compare risk across portfolios based on the threshold approach.

The positive difference, found for all portfolios, between average and median returns reflects the positive skewness of the distribution. Skewness of returns also tends to increase with decreasing market capitalization. Small and mid cap returns exhibit positive skewness, i.e., there is a greater-than-normal probability of a large positive return, in all the methods used to form portfolios. Moreover, small cap return skewness is always greater than large cap return skewness in our sample. The largest size portfolios display negative skewness in all the portfolios construction methods. Harvey and Siddique (2000) develop an asset pricing model which incorporates conditional skewness; they show that systematic skewness is economically important and commands a premium.[19]

(18) This could be evidence of positive size premium. However, these figures should be corrected by market risk and it is well known that smaller stocks tend to have higher betas than larger stocks.

(19) In the usual setup, investors are assumed to have preferences over the mean and the variance of portfolio returns. However, mean and variance alone cannot adequately characterize returns distributions. It is reasonable to assume that investors also have preferences for skewness. Large positive skewness means a high probability of a large positive return, and investors would surely favor this contingency. Thus, *ceteris paribus*, investors should prefer portfolios that are right-skewed to portfolios that are left-skewed.

Table 2 - Size-based portfolios returns

This table presents summary statistics of monthly percentage returns for size-based portfolios. Portfolios are formed using three methodologies: Panel A shows results for the threshold approach, Panel B for quartile-based portfolios, and Panel C for quintile-based portfolios. For each portfolio, the table reports the number of observations, the time-series arithmetic average of the value-weighted returns, the standard deviation of the average, the minimum return, the median return, the maximum return, the skewness coefficient, and the time-series geometric average of the value-weighted returns. The sample period covers December 31, 1998 through December 31, 2002.

	N. of obs.	Arithmetic average	Standard deviation	Min	Median	Max	Skewness	Geometric average
Panel A: Threshold approach								
Small cap	48	1.84	5.92	-10.98	0.89	27.00	1.58	1.68
Mid cap	48	1.44	5.38	-13.50	1.42	17.50	0.23	1.30
Large cap	48	0.54	6.62	-14.97	-0.02	14.63	-0.16	0.32
Panel B: Size quartiles								
1 (Smallest)	48	3.73	7.10	-6.26	2.32	25.24	1.44	3.50
2	48	2.41	6.14	-9.20	2.04	27.18	1.61	2.24
3	48	2.23	6.44	-10.50	0.63	24.98	1.19	2.04
4 (Largest)	48	0.70	6.33	-13.88	0.22	15.58	-0.11	0.50
Panel C: Size quintiles								
1 (Smallest)	48	3.92	8.15	-6.28	2.23	33.07	2.04	3.64
2	48	3.06	6.42	-7.71	2.66	26.34	1.33	2.87
3	48	2.24	6.79	-10.51	1.07	33.63	2.20	2.04
4	48	1.99	6.39	-10.67	0.42	26.47	1.37	1.80
5 (Largest)	48	0.68	6.35	-13.96	0.19	15.19	-0.13	0.48

Table 3 shows the correlation structure of returns. Three comments are in order. First, the correlation between small-size portfolios and large-size portfolios is well under one. Portfolio theory shows that we need to diversify over not perfectly correlated assets in order to reduce portfolio variance. Therefore, the imperfect correlation between small and large cap returns is first preliminary evidence supportive of the hypothesis that small caps could enhance portfolio diversification. Second, as expected, the correlation level depends on the size ranking methodology: the correlation between the largest size portfolio and the smallest size portfolio is 0.83 with the threshold approach, 0.47 with the quartile method, 0.52 with the quintile method. This implies that we need to take into account possible effects related to the method used to form portfolios. However, looking at portfolios correlation across different ranking methodologies we find reassuring evidence on this point. The third comment, therefore, relates to the consistency of the various methodologies used in forming portfolios: the correlation coefficients between the smallest portfolios, across different construction methods, are never lower than 0.71, and the correlation coefficients between the largest portfolios tend to unity. This means that portfolio returns constructed according to different methodologies turn out to behave similarly.

Table 3 - Returns correlation matrix

This table reports the contemporaneous correlation structure of size-based portfolio returns. The conversion table for portfolio symbols is reported below.

	S	M	L	4Q1	4Q2	4Q3	4Q4	5Q1	5Q2	5Q3	5Q4	5Q5
S	1	0.91	0.83	0.71	0.92	0.97	0.85	0.73	0.86	0.94	0.99	0.85
M		1	0.89	0.58	0.85	0.88	0.92	0.61	0.79	0.85	0.90	0.92
L			1	0.44	0.80	0.81	0.99	0.50	0.70	0.80	0.83	1.00
4Q1				1	0.71	0.70	0.47	0.90	0.80	0.73	0.68	0.47
4Q2					1	0.94	0.82	0.74	0.91	0.94	0.94	0.82
4Q3						1	0.83	0.74	0.87	0.95	0.99	0.83
4Q4							1	0.52	0.73	0.81	0.85	1.00
5Q1								1	0.71	0.78	0.69	0.52
5Q2									1	0.85	0.87	0.73
5Q3										1	0.94	0.81
5Q4											1	0.85
5Q5												1

Symbol	description
S, M, L	Small cap, mid cap, large cap portfolios
4Qx	x-th quartile portfolio
5Qx	x-th quintile portfolio

To provide a formal test of the hypothesis that investing in small caps enlarges the efficient frontier we perform a battery of spanning tests. As outlined by Huberman and Kandel (1987), the test for spanning is based on a multivariate regression relating the returns on N 'new' or 'test' assets as dependent variables to the returns to K benchmark assets as explanatory variables. The spanning hypothesis is rejected when the benchmark assets are not able to mimic the mean-variance characteristics of the new assets under investigation. This means that the new assets are autonomous asset classes relative to the benchmark assets.

Spanning tests, as they are linear restrictions on the estimated coefficients, can be performed by using likelihood ratio (LR), Wald (W), or Lagrange multiplier (LM) tests. In finite samples, it is well known that it exists a firm ranking of the three test statistics: $W \geq LR \geq LM$. Thus, to be more conservative in rejecting the null hypothesis of spanning we perform Lagrange multiplier tests.

Results for the LM test are reported in Table 4. In the first column of the table we indicate the test asset, i.e., the asset whose inclusion's effects are tested. The second column shows the set of benchmark assets, i.e., the original investment opportunity set that will eventually contain the new assets. The third and fourth columns display, respectively, the estimated intercept and the sum of the coefficients. The fifth column reports the adjusted-R^2 of the regression.

We find weak signal of no-spanning when looking at LM test results for the smallest size portfolios. In the three-portfolio classification (threshold approach), for the small cap portfolio spanning hypothesis is not rejected, for the mid cap portfolio spanning hypothesis is rejected at 1% level relative to an investment set made of both small and large caps, and for large cap stocks the hypothesis is rejected at 5% relative to a portfolio made of small and mid caps. In the size quartile and quintile rankings, only for the largest size portfolios spanning hypothesis is rejected at the usual confidence level relative to a set of benchmark assets that include all other portfolios constructed according to the same methodology.

Table 4 - Mean-variance spanning tests for size-based portfolios

This table reports the results of mean-variance spanning tests performed for size-based portfolios. Results are presented for the three methodologies to form size-based portfolios described in the body of the paper. In the table, Test asset indicates the asset to be included in the forming portfolio, and Benchmark assets indicates the assets already included in the portfolio. $\hat{\alpha}$, $\hat{\beta}'\iota$, and Adj-R^2 derive from the estimation of equation [1]. Mean VIF indicates the average Variance Inflation Factor computed for all dependent variables. Lagrange multiplier (LM) test refers to the joint hypothesis $H_0: \alpha = 0$ and $\beta'\iota = 1$.

Test asset	Benchmark assets	$\hat{\alpha}$	$\hat{\beta}'\iota$	Adj-R^2	Mean VIF	LM Test (p-value)
Panel A: Threshold approach						
S	M + L	0.42	0.95	0.73	5.92	0.86 *(0.65)*
M	S + L	0.53	0.85	0.89	2.51	10.33 *(0.00)*
L	S + M	-1.06	1.12	0.82	3.85	6.69 *(0.04)*
Panel B: Size quartiles						
4Q1	4Q2 + 4Q3 + 4Q4	1.12	0.79	0.61	7.56	4.64 *(0.10)*
4Q2	4Q1 + 4Q3 + 4Q4	0.07	0.96	0.91	3.37	0.45 *(0.80)*
4Q3	4Q1 + 4Q2 + 4Q4	0.14	1.03	0.90	3.65	0.81 *(0.67)*
4Q4	4Q1 + 4Q2 + 4Q3	-0.70	0.77	0.68	7.76	10.90 *(0.00)*
Panel C: Size quintiles						
5Q1	5Q2 + 5Q3 + 5Q4+ 5Q5	1.07	0.80	0.59	9.85	3.25 *(0.19)*
5Q2	5Q1 + 5Q3 + 5Q4+ 5Q5	0.83	0.93	0.87	8.97	4.94 *(0.08)*
5Q3	5Q1 + 5Q2 + 5Q4+ 5Q5	-0.35	1.04	0.93	5.64	2.14 *(0.34)*
5Q4	5Q1 + 5Q2 + 5Q3+ 5Q5	0.05	0.98	0.94	5.12	0.35 *(0.84)*
5Q5	5Q1 + 5Q2 + 5Q3+ 5Q4	-0.58	0.80	0.66	10.08	8.83 *(0.01)*

However, these results face two possible problems that have not been considered in the previous framework and need to be properly addressed. First, they only consider euro area stocks, and this is clearly a strong limitation. It is well known that portfolios need to be internationally diversified. Thus, it is much more realistic to adopt a global perspective and to include in our tests non-euro area asset classes too. Second, there is the possibility that results suffer from collinearity in the dependent variables used in the regressions. This problem is also related to the previous one: if we only consider euro area portfolios in the investment opportunity set, there is a higher probability of finding collinearity in the regressors matrix. To detect possible collinearity we report in Table 4 the average Variance Inflation Factors (VIFs) of all the regressors. Quintile-based portfolios are more subject to collinearity than quartile- and threshold-based portfolios. These latter, however, suffer from the effects of the portfolio construction method, that introduces noise in risk estimation and blurs portfolio return driving factors. The evidence of spanning for the small cap portfolio therefore can also be ascribed to the deflated variability induced by the large number of securities included in the portfolio.

In Table 5 we perform the same LM tests as those reported in Table 4 over an enlarged set of benchmark assets that considers the largest portfolio for each portfolio construction methodology, and also includes international asset classes.[20] We use publicly available indices to represent asset classes, easily available to the investors, different from euro area stocks: S&P500 for US large cap stocks, Russell 2000 for US small cap stocks, FTSE100 for UK large cap stocks, and MSCI Pacific for Asian companies.[21] This enables us to expressly consider the value of including euro area small caps in an internationally diversified portfolio, and to attenuate collinearity problems, as it can be seen from the comparison of the mean VIFs in Tables 4 and 5.

We find strong signal of rejection of the spanning hypothesis for all size-based portfolios but largest cap portfolios. It was somewhat expected that the hypothesis of spanning would not be rejected for largest stocks portfolios when considering an international set of benchmark assets. Other large cap portfolios were actually included in the investment opportunity set used for the spanning test (S&P500 and FTSE100), and thus the result is reasonably due to the presence of other asset classes with similar behavior. The interesting result relates to the small and mid caps evidence. Each size-based portfolio, largest cap excluded, behaves like an asset class, and this result holds true for all the methodologies used to form portfolios. The evidence is stronger for the second and fourth quintile portfolios: the rejection of the spanning hypothesis exhibits the largest statistical significance for those portfolios.

However, asymptotic tests (like Lagrange multiplier) are problematic to apply in finite samples and, especially for small size samples, test results could even be misleading. Thus, we use the finite sample distribution of the Wald test provided by Kan and Zhou (2001) to derive the following distribution for the Lagrange multiplier test under the null hypothesis of spanning:

$$\frac{\left(\frac{1-U}{U}\right)\left(\frac{T-K-1}{2}\right)}{1+\left(\frac{1-U}{U}\right)\left(\frac{T-K-1}{2T}\right)} \sim F_{2,T\text{-}K\text{-}1} \qquad [8]$$

where T is the length of the time-series, $U = \hat{\Sigma}/\tilde{\Sigma}$, $\hat{\Sigma}$ ($\tilde{\Sigma}$) the unconstrained (constrained) maximum likelihood estimator of the covariance matrix Σ.[22]

Results for the F-test version of the spanning test are also reported in Table 5, and

(20) We include the largest portfolio as it is the less correlated with the small cap portfolio.
(21) We employ euro-denominated versions of these returns to get rid of currency effects.
(22) This test also provides a good approximation for the non-normal case. The results of a simulation performed by Kan and Zhou (2001) show that the small sample distribution derived for the normal case works very well also when the disturbance is non-normal and homoskedastic.

are completely consistent with the previous ones both in terms of statistical significance, and in terms of displayed patterns across portfolios. In Table 6 we report the same tests as those reported in Table 5, performed over different sets of benchmark assets. Specifically, we test for the spanning of the smallest size portfolio relative to a set of benchmark assets that includes portfolios other than the largest size, already tested in Table 5. An interesting pattern arises by comparing Tables 5 and 6: as the size of the portfolios included in the benchmark asset set increases, the estimate of the intercept increases, the sum of the coefficients and the adjusted-R^2 decrease, and the probability of rejecting the spanning hypothesis increases. This pattern seems to capture the strongest underlying factors that drive size-based portfolios behavior. That is, when we consider more similar portfolios in terms of size, it is easier to replicate portfolio pay-offs.

Table 5 - Mean-variance spanning tests with international benchmark assets

This table reports the results of mean-variance spanning tests performed for size-based portfolios. The * in the Benchmark assets column indicates that, in addition to the listed portfolios, all the tests also consider the euro-denominated returns on the following indices: S&P500, Russell 2000, MSCI Pacific, and FTSE100. The F-test is performed according to [8] and, as the LM Test, refers to the joint hypothesis $H_0: \alpha = 0$ and $\beta' \iota = 1$.

Test Asset	Benchmark Assets	$\hat{\alpha}$	$\hat{\beta}'\iota$	Adj-R2	Mean VIF	LM Test (p-value)	F-test (p-value)
Panel A: Threshold approach							
S	L + *	0.98	0.44	0.75	4.57	24.49 *(0.00)*	21.87 *(0.00)*
M	L + *	0.85	0.66	0.87	4.57	24.45 *(0.00)*	21.80 *(0.00)*
L	*	0.68	1.03	0.73	4.13	2.00 *(0.37)*	0.94 *(0.40)*
Panel B: Size quartiles							
4Q1	4Q4 + *	2.79	0.16	0.35	4.54	21.74 *(0.00)*	17.38 *(0.00)*
4Q2	4Q4 + *	1.39	0.47	0.71	4.54	21.79 *(0.00)*	17.45 *(0.00)*
4Q3	4Q4 + *	1.14	0.59	0.78	4.54	18.65 *(0.00)*	13.34 *(0.00)*
4Q4	*	0.79	0.97	0.73	4.13	3.04 *(0.22)*	1.45 *(0.25)*
Panel C: Size quintiles							
5Q1	5Q5 + *	2.98	0.20	0.26	4.55	17.01 *(0.00)*	11.53 *(0.00)*
5Q2	5Q5 + *	1.98	0.40	0.65	4.55	22.85 *(0.00)*	19.08 *(0.00)*
5Q3	5Q5 + *	1.15	0.49	0.66	4.55	16.34 *(0.00)*	10.84 *(0.00)*
5Q4	5Q5 + *	0.91	0.57	0.80	4.55	19.43 *(0.00)*	14.28 *(0.00)*
5Q5	*	0.79	0.98	0.73	4.13	2.91 (0.23)	1.39 (0.26)

Spanning tests also have an economic interpretation. de Roon and Nijman (2001) and Kan and Zhou (2001) decompose the spanning test in two parts: one related to the tangency portfolio, and the other to the global minimum variance portfolio. The first part of the spanning tests measures the changes in the Sharpe ratio occurring when the test assets are added to the set of benchmark assets; that means to test if the new tangency portfolio has zero weights in the test assets. This part of the test refers to the restriction on the intercepts of the regression ($\underline{a} = \underline{0}_N$). The second part of the spanning test measures the change in the global minimum variance portfolio; i.e., it is a test of whether the global minimum variance portfolio has zero weights in the test assets. This part of the test refers to the constraint on the estimated coefficients ($\mathrm{B}\underline{\iota}_K = \underline{\iota}_N$).

Table 6 - Mean-variance spanning tests with different benchmark assets

This table reports the results of mean-variance spanning tests for size-based portfolios. The tests are the same as those reported in Table 5, performed over a different set of benchmark assets. For other definitions refer to the previous tables.

Test asset	Benchmark assets	$\hat{\alpha}$	$\hat{\beta}'\iota$	Adj-R2	Mean VIF	LM Test (p-value)	F-test (p-value)
Panel A: Threshold approach							
S	M + *	0.26	0.74	0.79	4.25	6.77	3.45
						(0.03)	*(0.04)*
Panel B: Size quartiles							
4Q1	4Q2 + *	1.06	0.74	0.70	4.13	5.27	2.59
						(0.07)	*(0.09)*
4Q1	4Q3 + *	1.65	0.54	0.55	4.13	10.10	5.60
						(0.01)	*(0.01)*
Panel C: Size quintiles							
5Q1	5Q2 + *	0.73	0.83	0.56	4.12	1.28	0.58
						(0.53)	*(0.57)*
5Q1	5Q3 + *	1.39	0.81	0.69	4.01	4.96	2.42
						(0.08)	*(0.10)*
5Q1	5Q4 + *	1.77	0.69	0.49	4.24	5.56	2.75
						(0.06)	*(0.07)*

In Table 7 we perform this decomposition of the spanning test in order to investigate the sources of the spanning hypothesis rejection. Following Kan and Zhou (2001) we implement a step-down procedure to test the spanning hypothesis; i.e., we run a sequential test. First, we test $\underline{a} = \underline{0}_N$, and then we test $\mathrm{B}\underline{\iota}_K = \underline{\iota}_N$ conditional on the constraint $\underline{a} = \underline{0}_N$. We find that both the tangency portfolio and the global minimum variance portfolio can be improved by the inclusion of euro area size-sorted portfolios. However, the evidence is stronger in favor of the enhancement of the tangency portfolio. When also including international asset classes in the benchmark asset set, the null hypothesis related to the global minimum variance portfolio is no more rejected in some cases at the usual confidence levels (e.g., the third quartile portfolio at 5%), whereas test results still consistently reject the null hypothesis for the tangency portfolio at the lowest possible level of confidence for all size-sorted portfolios, but the largest cap portfolios.

This evidence can also be examined in light of the aforementioned economic interpretation of the spanning test. The first part of the test ($\underline{a} = \underline{0}_N$) measures whether there is an improvement of the tangency portfolio's characteristics or, alternatively stated, whether there is intersection at the most extreme points of the frontier (de Roon and

Nijman, 2001).[23] Therefore, this result is particularly important for investors with a weak risk aversion, as they will benefit more from the improvement of the tangency portfolio's characteristics.

Table 7 - Step-down mean-variance spanning tests

This table reports two mean-variance spanning tests for size-based portfolios. The first test (F_1) is an *F*-test of $H_0 : \alpha = 0$, and the second test (F_2) is an *F*-test of $H_0 : \beta' \iota = 1$ conditional on $\alpha = 0$. Results are presented for threshold approach (*Panel A*), quartile-based (*Panel B*), and quintile-based (*Panel C*) portfolios. For other definitions refer to the previous tables.

Test asset	Benchmark asset	F_1-test		F_2-test	
		Statistic	*p*-value	Statistic	*p*-value
Panel A: Threshold approach					
S	L + *	35.96	0.00	4.86	0.03
M	L + *	31.53	0.00	8.54	0.01
L	*	0.10	0.76	1.83	0.18
Panel B: Size quartiles					
4Q1	4Q4 + *	21.98	0.00	10.37	0.00
4Q2	4Q4 + *	24.87	0.00	7.80	0.01
4Q3	4Q4 + *	18.84	0.00	6.13	0.02
4Q4	*	0.10	0.75	2.69	0.11
Panel C: Size quintiles					
5Q1	5Q5 + *	13.45	0.00	7.90	0.01
5Q2	5Q5 + *	23.62	0.00	11.77	0.00
5Q3	5Q5 + *	16.69	0.00	3.66	0.06
5Q4	5Q5 + *	22.45	0.00	4.42	0.04
5Q5	*	0.06	0.81	2.63	0.11

We now turn to the second part of our empirical analysis, i.e., to the measurement of potential diversification gains attached to investing in size-based portfolios. To compute ψ, λ, and δ as, respectively, in equations [3], [4], and [5] we use historical sample estimates of risk and expected return as inputs to a standard Markowitz-inspired mean-variance optimization model.[24]

Policy constraints, i.e., lower and upper bounds in portfolio weights, play an important role in the implementation of an asset allocation model. To assess the impact of such restrictions on our results, we employ (and compare the effects of) three sets of policy constraints:

(23) As shown in sub-Section 3.1, intersection refers to the case when the frontier of the benchmark assets and the frontier generated from the enlarged investment opportunity set (benchmark and new assets) have exactly one point in common. This condition affects only investors with a specific mean-variance utility function, i.e., the one that identifies this special portfolio as the optimal portfolio.

(24) In the optimization process, the monthly risk-free rate is assumed to be zero. It is well know that mean-variance optimization suffers from two main problems: poor out-of-sample performance of the optimal portfolio, and instability of the optimal portfolio's weights. Both issues pertain to the general question of the estimation risk in the practical application of mean-variance optimization. With a positive risk-free rate, the optimal portfolio would have an even higher return *per* unit of risk than when assuming a zero rate of interest, and any undesirable characteristic of the tangency portfolio would be accentuated. Thus, as Jorion (1985) points out, the zero risk-free rate assumption with monthly returns reduces the effects of the estimation risk.

$$\sum_{i+1}^{K+N} w_i = 1 \text{ and } -1 \leq w_i \leq 1 \text{ ('unconstrained' policy)} \qquad [9a]$$

$$\sum_{i+1}^{K+N} w_i = 1 \text{ and } 0 \leq w_i \leq 1 \text{ ('no-short sales' policy)} \qquad [9b]$$

$$\sum_{i+1}^{K+N} w_i = 1 \text{ and } 0 \leq w_i \leq 0.5 \text{ ('upper bound' policy)} \qquad [9c]$$

The 'unconstrained' policy simply states that the sum of the proportions invested in each asset adds to one, and that all assets can be purchased or short sold up to 100 percent of the total value of the portfolio. However, when running portfolio optimization programs with unconstrained weights, mean-variance efficient portfolios often contain large long and short positions in a number of assets. Large short positions are difficult to implement in practice. This is particularly true in case of small and mid cap stocks: short sales could be too costly for most of the stocks included in our sample. Therefore, the 'no short sales' policy forces portfolio weights to be non-negative.

The third policy considered here ('upper bound') defines a maximum percentage of the portfolio that can be invested in a single asset. We set 0.5 as upper bound in the optimization process. In the asset management industry upper and/or lower limits are frequently imposed in solving optimal portfolio problems, either for regulatory reasons or for asset managers subjective decisions. Empirical findings suggest that imposing constraints on portfolio weights, with no short selling included, reduces the estimation risk and improves the efficiency of optimal portfolios constructed using sample moments (Frost and Savarino, 1988).

Table 8 separately reports empirical measures of diversification gains for the three sets of policy constraints previously defined. We find large improvements in the efficient frontier when we move from an investment opportunity set that only considers euro area large cap stocks and international assets (the one denoted by '4Q4 + *' in Table 8) to an investment opportunity set that includes, in addition to the international assets, all euro area size-sorted portfolios (the one indicates as '4Q4 + 4Q3 + 4Q2 + 4Q1 + *' in Table 8). Such improvements have been measured along two dimensions. First, we look at the riskiness of the global minimum variance (GMV) portfolio. For the quartile (quintile) decomposition, we find that the standard deviation of the GMV portfolio decreases of about 18% (16%)[25] in the 'unconstrained' optimization policy, 10% (8%) in the 'no-short sales' policy, and 11% (9%) in the 'upper bound' policy.

Second, we document the change in the Sharpe ratio, that corresponds to the shift in the optimal risky portfolio when the test asset is added to the set of benchmark assets. From Table 8 it can be seen that, when all euro area size-sorted portfolios are included in the investment opportunity set, the Sharpe ratio increases according to a factor that ranges from 3.9 (0.616 relative to 0.156), in the 'unconstrained' policy for the quartile portfolios,

(25) This is the percentage change from 4.68% (4.68%) to 3.83% (3.93%) in the standard deviation of the GMV portfolio for the quartile (quintile) decomposition.

to 6.9 (0.522 relative to 0.076), in the 'upper bound' policy for the quintile portfolios. The largest changes in the Sharpe ratio, for the quintile decomposition, relate to the introduction of the second quintile (5Q2) and fourth quintile (5Q4) portfolios. This evidence matches with the spanning test results reported in Table 5.

Table 8 - Diversification gains

This table reports the ex post measures of diversification gains. For the definition of ψ and λ refer, respectively, to [3] and [4] in the body of the paper. GMV indicates the standard deviation of the global minimum variance portfolio, and SR indicates the Sharpe ratio of the tangency portfolio. Results are separately reported for the three sets of policy constraints defined in [9]. The * in the Investment opportunity set column indicates that, in addition to the listed portfolios, the optimization also considers the following indices: S&P500, Russell 2000, MSCI Pacific, and FTSE100.

	Unconstrained		No short sales		Upper bound	
Investment opportunity set	GMV (ψ)	SR (λ)	GMV (ψ)	SR (λ)	GMV (ψ)	SR (λ)
Panel A: Size quartiles						
4Q4 + *	4.68%	0.156	4.85%	0.111	4.97%	0.077
4Q4 + 4Q3 + *	4.13%	0.379	4.66%	0.350	4.72%	0.243
	(0.55%)	*(0.223)*	*(0.19%)*	*(0.239)*	*(0.25%)*	*(0.166)*
4Q4 + 4Q3 + 4Q2 + *	3.87%	0.537	4.54%	0.396	4.56%	0.378
	(0.26%)	*(0.158)*	*(0.12%)*	*(0.046)*	*(0.16%)*	*(0.135)*
4Q4 + 4Q3 + 4Q2 + 4Q1 + *	3.83%	0.616	4.39%	0.531	4.43%	0.499
	(0.04%)	*(0.079)*	*(0.15%)*	*(0.135)*	*(0.13%)*	*(0.121)*
Panel B: Size quintiles						
5Q5 + *	4.68%	0.154	4.85%	0.108	4.97%	0.076
5Q5+ 5Q4 + *	4.05%	0.347	4.65%	0.315	4.70%	0.222
	(0.53%)	*(0.193)*	*(0.20%)*	*(0.207)*	*(0.27%)*	*(0.146)*
5Q5 + 5Q4+ 5Q3 + *	4.03%	0.441	4.61%	0.334	4.64%	0.329
	(0.02%)	*(0.094)*	*(0.04%)*	*(0.019)*	*(0.06%)*	*(0.107)*
5Q5 + 5Q4+ 5Q3 + 5Q2 + *	3.95%	0.580	4.54%	0.482	4.58%	0.414
	(0.08%)	*(0.139)*	*(0.07%)*	*(0.148)*	*(0.06%)*	*(0.85)*
5Q5 + 5Q4+ 5Q3 + 5Q2 + 5Q1 + *	3.93%	0.682	4.48%	0.524	4.52%	0.522
	(0.02%)	*(0.102)*	*(0.06%)*	*(0.042)*	*(0.06%)*	*(0.108)*

These results are also consistent with the statistical significance analysis reported in Table 7, and with a graphical representation of the δ metric, provided in Figure 3. This metric measures the differential return earned over a portfolio with the same level of risk. As expected, and as evident in Figure 3, results depend on the set of policy constraints used in the portfolio optimization process jointly with the level of risk considered and the composition of the investment opportunity set. For example, if we consider a 6% monthly standard deviation, the inclusion of all euro area size-sorted portfolios relative to an investment opportunity set that only includes the largest quintile portfolio provides an increase in portfolio return approximately equal to 3.3% in the 'unconstrained' policy, 2.4% in the 'no-short sales' policy, and 2.4% in the 'upper bound' policy.

Holding all other factors constant, if we consider the inclusion of the two smallest quintile portfolios in an investment set that already includes quintile portfolios one to three, the increase in portfolio return is about 1.6% in in the 'unconstrained' policy, 1.1% in the 'no-short sales' policy, and 1.1% in the 'upper bound' policy. Looking at Figure 3 we also find that the largest increases in portfolio return coincide with the inclusion in the investment set of quintile 2 and quintile 4 portfolios. This result is consistent with the statistical significance analysis previously performed where the rejection of the spanning hypothesis was stronger for the second and the fourth quintile portfolios (see Table 5).

Figure 3 - Ex-post portfolio differential returns

This figure provides graphical evidence of δ, i.e., the ex-post differential return computed over different investment opportunity sets holding constant portfolio's risk. Figure 3a refers to the unconstrained optimization process, Figure 3b to the case where short sales are not allowed, Figure 3c to the case where the maximum weight *per* asset class is 0.5.

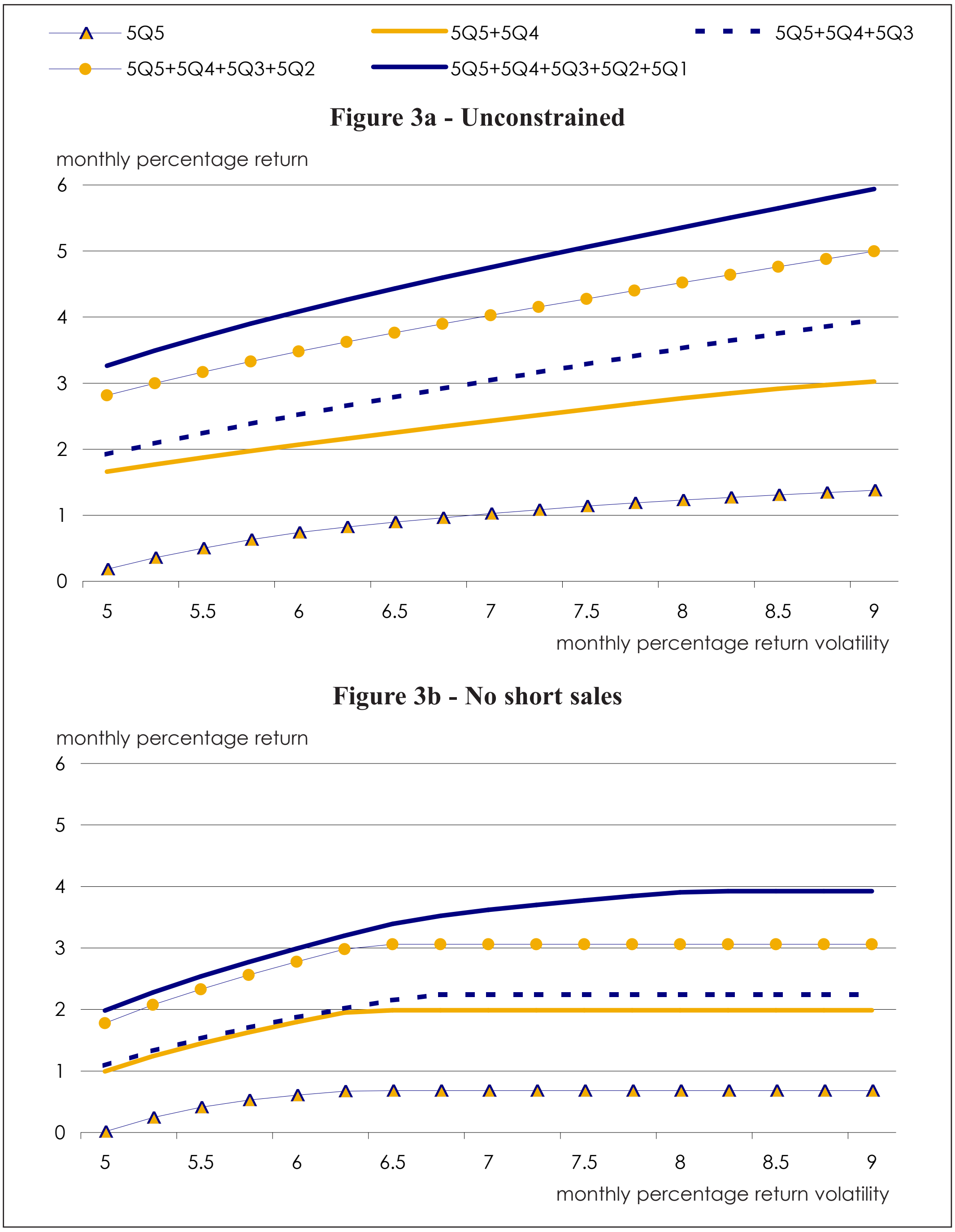
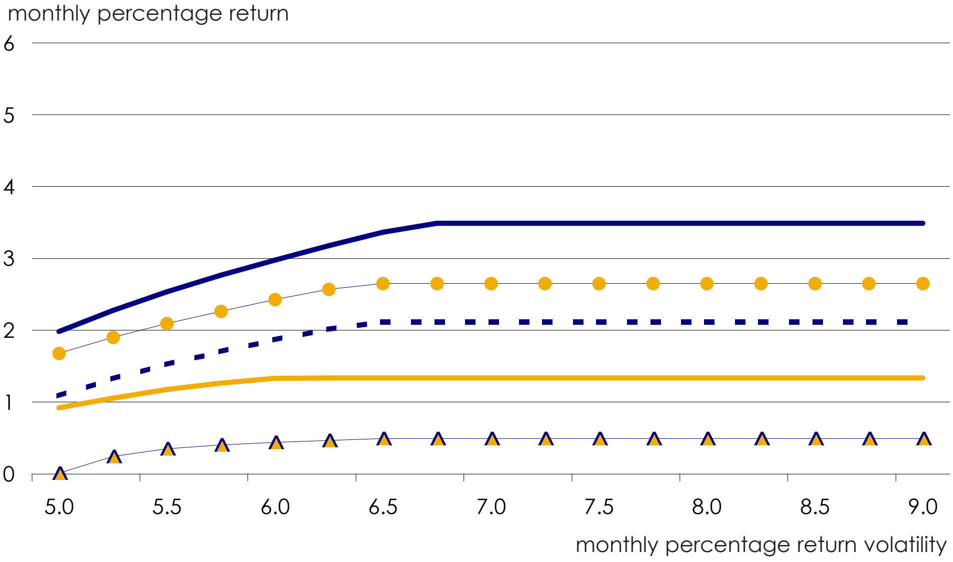


Figure 3c - Upper bound



In evaluating these results, however, care must be taken for two reasons. First, a potential drawback of the empirical measurement of diversification benefits is in the use of paper portfolios that ignore transaction costs associated with investments in small and mid cap stocks. It should be noted nonetheless that the actual impact on portfolio return of these costs depends upon the length of the investment horizon. As the holding period increases, the amortized cost of transacting decreases and thus the impact of transaction costs on portfolio returns declines.[26] Second, by being ex post in nature, the metrics computed to measure diversification gains ignore the issue of estimation risk, or parameter uncertainty, and therefore may have mismeasured the truly realizable portion of the potential gains from size diversification. This is a common drawback for all the implementations of the standard mean-variance analysis.

(26) Chalmers and Kadlec (1998) propose a version of the bid-ask spread (defined as 'amortized spread') that accounts for *average realized* holding periods. They provide an empirical examination of the amortized spread and find stronger evidence that amortized spread is priced in stock returns than quoted spread is.

6. Summary and conclusions

Diversification is the only free lunch finance does offer (Campbell, 2000). Portfolio theory shows that diversifying over not perfectly correlated assets can improve investor welfare by reducing portfolio variance. To fully take advantage of diversification opportunities, an investor has to identify all asset classes available for investing. Thus, the basic question we address in this paper is whether euro area small cap stocks are an asset class, i.e., if there is any benefit – from a portfolio efficiency perspective – in investing in euro area small caps.

We apply a mean-variance spanning approach to test for the existence of diversification benefits related to investing in euro area small cap stocks and estimate the dimension of diversification gains using several empirical methodologies. The results of our study show that important gains in portfolio efficiency can be achieved through size diversification, i.e., an asset allocation that invests in different size stocks. Spanning tests indicate that both small and mid cap euro area stocks, as classified by size quartile and quintile rankings, arise as truly autonomous asset classes. The rejection of the spanning hypothesis exhibits the largest statistical significance for the second and the fourth quintile portfolios. Results are robust to different methodologies used to form size-based portfolios and to the inclusion of international asset classes in the investment opportunity set. Interestingly, euro area small and mid caps are asset classes also when portfolio holdings already include US small and large cap stocks.

We also investigate the source of the diversification benefits related to investing in euro area small and mid cap stocks. We find that the improvement in the efficient frontier originates from an upward movement of both the tangency portfolio and the global minimum variance portfolio. The evidence is stronger in favor of the enhancement of the tangency portfolio. This result is particularly important for investors with low risk aversion, as they would benefit more from the enlargement of the minimum variance frontier.

This study has two practical implications for portfolio management. First, market capitalization arises as an important variable in the construction of long term portfolio weights, i.e., in the definition of the benchmark portfolio, as it significantly affects stock return characteristics. Second, our results show that euro area small and mid cap stocks behave differently from euro area large cap stocks, as well as other international asset classes. This implies that it is useful to expressly consider different euro area size-ranked portfolios in the strategic asset allocation process.

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